Indian Hill High School
AP Calculus AB

Calculus AB is primarily concerned with developing the students' understanding of the concepts of calculus and providing experience with its methods and applications. The course emphasizes a multi-representational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations also are important.

Broad concepts and widely applicable methods are emphasized. The focus of the course is neither manipulation nor memorization of an extensive taxonomy of functions, curves, theorems, or problem types. Thus, although facility with manipulation and computational competence are important outcomes, they are not the core of this course.

Technology should be used regularly by students and teachers to reinforce the relationships among the multiple representation of functions, to confirm written work, to implement experimentation, and to assist in interpreting results.

Through the use of unifying themes of derivatives, integrals, limits, approximation, and applications and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics. These themes are developed using all the functions listed in the prerequisites.

Goals:

Students should be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.

Students should understand the meaning of the derivative in terms of a rate of change and local linear approximation and should be able to use derivatives to solve a variety of problems.

Students should understand the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of a rate of change and should be able to use integrals to solve a variety of problems.

Students should understand the relationship between the derivative and the definite integral as expressed in both parts of the fundamental Theorem of Calculus.

Student should be able to communicate mathematics both orally and in well-written sentences and should be able to explain solutions to problems.

Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral.

Students should be able to use technology to help solve problems, experiment, interpret results, and verify conclusions.

Students should be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.

Students should develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

Topic Outline

This outline of topics is intended to indicate the scope of the course, but it is not necessarily the order in which the topics are to be taught. Teachers may find that topics are best taught in different orders.

I. Functions, Graphs, and Limits

Analysis of graphs. With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions (including one-sided limits).

An intuitive understanding of the limiting process.
Calculating limits using algebra.
Estimating limits from graphs or tables of data.

Asymptotic and unbounded behavior.

Understanding asymptotes in terms of graphical behavior.
Describing asymptotic behavior in terms of limits involving infinity.
Comparing relative magnitudes of functions and their rates of change. (For example, contrasting exponential growth, polynomial growth, and logarithmic growth.)

Continuity as a property of functions.

An intuitive understanding of continuity. (Close values of the domain lead to close values of the range.)
Understanding continuity in terms of limits.
Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

II. Derivatives

Concept of derivative.

Derivative presented graphically, numerically, and analytically.
Derivative interpreted as an instantaneous rate of change.
Derivative defined as the limit of the difference quotient.
Relationship between differentiability and continuity.

Derivative at a point.

Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
Tangent line to a curve at a point and local linear approximation.
Instantaneous rate of change as the limit of average rate of change.
Approximate rate of change from graphs and tables of values.

Derivative as a function.

Corresponding characteristics of graphs of f and f'.
Relationship between the increasing and decreasing behavior of f and the sign of f'.
The Mean Value Theorem and its geometric consequences.
Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second Derivatives.

Corresponding characteristics of the graphs of f, f', and f''.
Relationship between the concavity of f and the sign of f''.
Points of inflection as places where concavity changes.

Applications of derivatives.

Analysis of curves, including the notions of monotonicity and concavity.
Optimization, both absolute (global) and relative (local) extrema.
Modeling rates of change, including related rates problems.
Use of implicit differentiation to find the derivative of an inverse function.
Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.

Computation of derivatives.

Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
Basic rules for the derivative of sums, products, and quotients of functions.
Chain rule and implicit differentiation.

III. Integrals

Interpretation and properties of definite integrals.

Computation of Riemann sums using left, right, and midpoint evaluation points.
Definite integral as a limit of Riemann sums over equal subdivisions.
Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

Basic properties of definite integrals. (Examples include additivity and linearity.)

Applications of integrals. Appropriate integrals are used in a variety of applications to model physical, biological, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the integral of a rate of change to give accumulated change or using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, and the distance traveled by a particle along a line.